

11

Angular Momentum

CHAPTER OUTLINE

- 11.1 The Vector Product and Torque
- 11.2 Angular Momentum
- 11.3 Angular Momentum of a Rotating Rigid Object
- 11.4 Conservation of Angular Momentum
- 11.5 The Motion of Gyroscopes and Tops
- 11.6 Angular Momentum as a Fundamental Quantity

ANSWERS TO QUESTIONS

- Q11.1** No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis.
- Q11.2** $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ is a scalar quantity, since $(\mathbf{B} \times \mathbf{C})$ is a vector. Since $\mathbf{A} \cdot \mathbf{B}$ is a scalar, and the cross product between a scalar and a vector is not defined, $(\mathbf{A} \cdot \mathbf{B}) \times \mathbf{C}$ is undefined.
- Q11.3** (a) Down-cross-left is away from you: $-\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) = -\hat{\mathbf{k}}$
- (b) Left-cross-down is toward you: $-\hat{\mathbf{i}} \times (-\hat{\mathbf{j}}) = \hat{\mathbf{k}}$

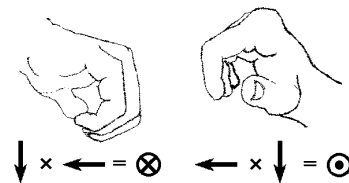


FIG. Q11.3

- Q11.4** The torque about the point of application of the force is zero.
- Q11.5** You cannot conclude anything about the magnitude of the angular momentum vector without first defining your axis of rotation. Its direction will be perpendicular to its velocity, but you cannot tell its direction in three-dimensional space until an axis is specified.
- Q11.6** Yes. If the particles are moving in a straight line, then the angular momentum of the particles about any point on the path is zero.
- Q11.7** Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.
- Q11.8** No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

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- Q11.9** There must be two rotors to balance the torques on the body of the helicopter. If it had only one rotor, the engine would cause the body of the helicopter to swing around rapidly with angular momentum opposite to the rotor.
- Q11.10** The angular momentum of the particle about the center of rotation is constant. The angular momentum about any point that does not lie along the axis through the center of rotation and perpendicular to the plane of motion of the particle is not constant in time.
- Q11.11** The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.
- Q11.12** The diver leaves the platform with some angular momentum about a horizontal axis through her center of mass. When she draws up her legs, her moment of inertia decreases and her angular speed increases for conservation of angular momentum. Straightening out again slows her rotation.
- Q11.13** Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.
- Q11.14** The angular speed must increase. Since gravity does not exert a torque on the system, its angular momentum remains constant as the gas contracts.
- Q11.15** Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases.
- Q11.16** The turntable will rotate counterclockwise. Since the angular momentum of the mouse-turntable system is initially zero, as both are at rest, the turntable must rotate in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero.
- Q11.17** Since the cat cannot apply an external torque to itself while falling, its angular momentum cannot change. Twisting in this manner changes the orientation of the cat to feet-down without changing the total angular momentum of the cat. Unfortunately, humans aren't flexible enough to accomplish this feat.
- Q11.18** The angular speed of the ball must increase. Since the angular momentum of the ball is constant, as the radius decreases, the angular speed must increase.
- Q11.19** Rotating the book about the axis that runs across the middle pages perpendicular to the binding—most likely where you put the rubber band—is the one that has the intermediate moment of inertia and gives unstable rotation.
- Q11.20** The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

SOLUTIONS TO PROBLEMS

Section 11.1 The Vector Product and Torque

$$\text{P11.1} \quad \mathbf{M} \times \mathbf{N} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \boxed{-7.00\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}} - 10.0\hat{\mathbf{k}}}$$

$$\text{P11.2} \quad (\text{a}) \quad \text{area} = |\mathbf{A} \times \mathbf{B}| = AB \sin \theta = (42.0 \text{ cm})(23.0 \text{ cm}) \sin(65.0^\circ - 15.0^\circ) = \boxed{740 \text{ cm}^2}$$

$$\begin{aligned} (\text{b}) \quad \mathbf{A} + \mathbf{B} &= [(42.0 \text{ cm}) \cos 15.0^\circ + (23.0 \text{ cm}) \cos 65.0^\circ] \hat{\mathbf{i}} + [(42.0 \text{ cm}) \sin 15.0^\circ + (23.0 \text{ cm}) \sin 65.0^\circ] \hat{\mathbf{j}} \\ \mathbf{A} + \mathbf{B} &= (50.3 \text{ cm}) \hat{\mathbf{i}} + (31.7 \text{ cm}) \hat{\mathbf{j}} \\ \text{length} = |\mathbf{A} + \mathbf{B}| &= \sqrt{(50.3 \text{ cm})^2 + (31.7 \text{ cm})^2} = \boxed{59.5 \text{ cm}} \end{aligned}$$

$$\text{P11.3} \quad (\text{a}) \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\hat{\mathbf{k}}}$$

$$\begin{aligned} (\text{b}) \quad |\mathbf{A} \times \mathbf{B}| &= |\mathbf{A}| |\mathbf{B}| \sin \theta \\ 17 &= 5\sqrt{13} \sin \theta \\ \theta &= \arcsin\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6^\circ} \end{aligned}$$

$$\text{P11.4} \quad \mathbf{A} \cdot \mathbf{B} = -3.00(6.00) + 7.00(-10.0) + (-4.00)(9.00) = -124$$

$$AB = \sqrt{(-3.00)^2 + (7.00)^2 + (-4.00)^2} \cdot \sqrt{(6.00)^2 + (-10.0)^2 + (9.00)^2} = 127$$

$$(\text{a}) \quad \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168^\circ}$$

$$(\text{b}) \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 12.0\hat{\mathbf{k}}$$

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{(23.0)^2 + (3.00)^2 + (-12.0)^2} = 26.1$$

$$\sin^{-1}\left(\frac{|\mathbf{A} \times \mathbf{B}|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9^\circ} \text{ or } 168^\circ$$

$$(\text{c}) \quad \text{Only } \boxed{\text{the first method}} \text{ gives the angle between the vectors unambiguously.}$$

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*P11.5 $\tau = \mathbf{r} \times \mathbf{F} = 0.450 \text{ m}(0.785 \text{ N})\sin(90^\circ - 14^\circ) \text{ up} \times \text{east}$
 $= \boxed{0.343 \text{ N} \cdot \text{m north}}$

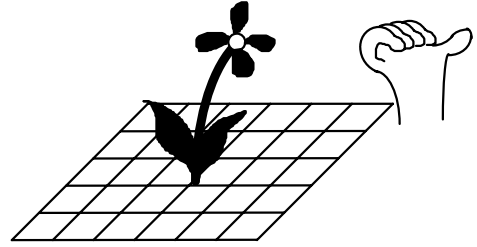


FIG. P11.5

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$?

$$8 - 9 - 4 = -5 \neq 0$$

No. The cross product could not work out that way.

P11.7 $|\mathbf{A} \times \mathbf{B}| = \mathbf{A} \cdot \mathbf{B} \Rightarrow AB \sin \theta = AB \cos \theta \Rightarrow \tan \theta = 1$ or

$\theta = \boxed{45.0^\circ}$

P11.8 (a) $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0 - 0) - \hat{\mathbf{j}}(0 - 0) + \hat{\mathbf{k}}(2 - 9) = \boxed{(-7.00 \text{ N} \cdot \text{m})\hat{\mathbf{k}}}$

(b) The particle's position vector relative to the new axis is $1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{j}} = 1\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

$$\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0 \text{ N} \cdot \text{m})\hat{\mathbf{k}}}$$

P11.9 $\boxed{|\mathbf{F}_3| = |\mathbf{F}_1| + |\mathbf{F}_2|}$

The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance $\mathbf{OD}$, therefore translating the point of application of $\mathbf{F}_3$ to any other point along

$\mathbf{BC}$ will not change the net torque.

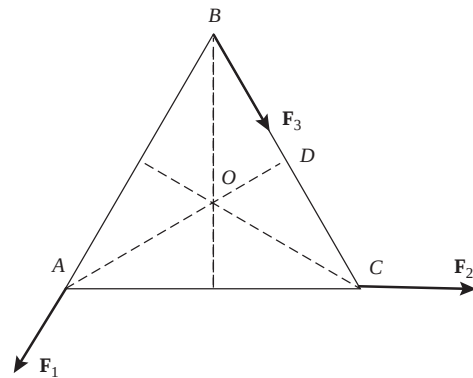


FIG. P11.9

***P11.10** $|\hat{\mathbf{i}} \times \hat{\mathbf{i}}| = 1 \cdot 1 \cdot \sin 0^\circ = 0$

$\hat{\mathbf{j}} \times \hat{\mathbf{j}}$ and $\hat{\mathbf{k}} \times \hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$$|\hat{\mathbf{i}} \times \hat{\mathbf{j}}| = 1 \cdot 1 \cdot \sin 90^\circ = 1$$

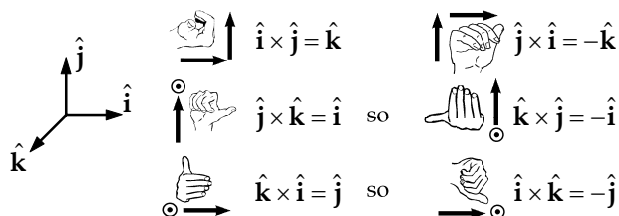


FIG. P11.10

Section 11.2 Angular Momentum

P11.11 $L = \sum m_i v_i r_i$
 $= (4.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m}) + (3.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m})$

$L = 17.5 \text{ kg} \cdot \text{m}^2/\text{s}$, and

$$\mathbf{L} = (17.5 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\mathbf{k}}$$

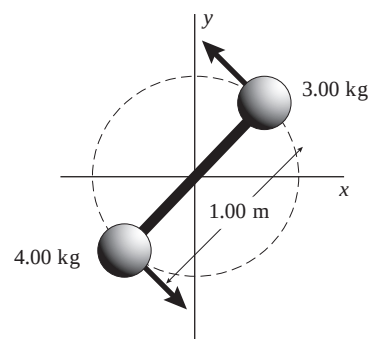


FIG. P11.11

P11.12 $\mathbf{L} = \mathbf{r} \times \mathbf{p}$
 $\mathbf{L} = (1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}) \text{ m} \times (1.50 \text{ kg})(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}) \text{ m/s}$
 $\mathbf{L} = (-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}) \text{ kg} \cdot \text{m}^2/\text{s} = \boxed{(-22.0 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\mathbf{k}}}$

P11.13 $\mathbf{r} = (6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}) \text{ m}$ $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 5.00\hat{\mathbf{j}} \text{ m/s}$

so $\mathbf{p} = m\mathbf{v} = 2.00 \text{ kg}(5.00\hat{\mathbf{j}} \text{ m/s}) = 10.0\hat{\mathbf{j}} \text{ kg} \cdot \text{m/s}$

and $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{(60.0 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\mathbf{k}}}$

P11.14 $\sum F_x = ma_x$ $T \sin \theta = \frac{mv^2}{r}$

$\sum F_y = ma_y$ $T \cos \theta = mg$

So $\frac{\sin \theta}{\cos \theta} = \frac{v^2}{rg}$ $v = \sqrt{rg \frac{\sin \theta}{\cos \theta}}$

$L = rmv \sin 90.0^\circ$

$L = rm \sqrt{rg \frac{\sin \theta}{\cos \theta}}$

$L = \sqrt{m^2 g r^3 \frac{\sin \theta}{\cos \theta}}$

$r = \ell \sin \theta$, so

$L = \sqrt{m^2 g \ell^3 \frac{\sin^4 \theta}{\cos \theta}}$

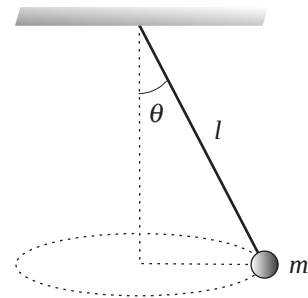


FIG. P11.14

P11.15 The angular displacement of the particle around the circle is $\theta = \omega t = \frac{vt}{R}$.

The vector from the center of the circle to the mass is then $R \cos \theta \hat{\mathbf{i}} + R \sin \theta \hat{\mathbf{j}}$.

The vector from point P to the mass is

$$\mathbf{r} = R\hat{\mathbf{i}} + R \cos \theta \hat{\mathbf{i}} + R \sin \theta \hat{\mathbf{j}}$$

$$\mathbf{r} = R \left[\left(1 + \cos \left(\frac{vt}{R} \right) \right) \hat{\mathbf{i}} + \sin \left(\frac{vt}{R} \right) \hat{\mathbf{j}} \right]$$

The velocity is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -v \sin \left(\frac{vt}{R} \right) \hat{\mathbf{i}} + v \cos \left(\frac{vt}{R} \right) \hat{\mathbf{j}}$$

So $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$

$$\mathbf{L} = mvR \left[(1 + \cos \omega t) \hat{\mathbf{i}} + \sin \omega t \hat{\mathbf{j}} \right] \times \left[-\sin \omega t \hat{\mathbf{i}} + \cos \omega t \hat{\mathbf{j}} \right]$$

$$\mathbf{L} = \boxed{mvR\hat{\mathbf{k}} \left[\cos \left(\frac{vt}{R} \right) + 1 \right]}$$

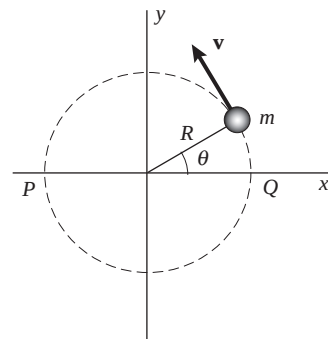


FIG. P11.15

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\tau| = |\mathbf{r} \times \mathbf{F}| = 8.00 \times 10^{-2} \text{ m} (4.00 \text{ kg}) (9.80 \text{ m/s}^2) = \boxed{3.14 \text{ N} \cdot \text{m}}.$$

(b) $|\mathbf{L}| = |\mathbf{r} \times m\mathbf{v}| + I\omega$ $|\mathbf{L}| = Rmv + \frac{1}{2}MR^2 \left(\frac{v}{R} \right) = R \left(m + \frac{M}{2} \right) v = \boxed{(0.400 \text{ kg} \cdot \text{m})v}$

(c) $\tau = \frac{dL}{dt} = (0.400 \text{ kg} \cdot \text{m})a$ $a = \frac{3.14 \text{ N} \cdot \text{m}}{0.400 \text{ kg} \cdot \text{m}} = \boxed{7.85 \text{ m/s}^2}$

P11.17 (a) zero

(b) At the highest point of the trajectory,

$$x = \frac{1}{2} R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and}$$

$$y = h_{\max} = \frac{(v_i \sin \theta)^2}{2g}$$

$$\begin{aligned} \mathbf{L}_1 &= \mathbf{r}_1 \times m\mathbf{v}_1 \\ &= \left[\frac{v_i^2 \sin 2\theta}{2g} \hat{\mathbf{i}} + \frac{(v_i \sin \theta)^2}{2g} \hat{\mathbf{j}} \right] \times mv_{xi} \hat{\mathbf{i}} \\ &= \left[\frac{-m(v_i \sin \theta)^2 v_i \cos \theta}{2g} \hat{\mathbf{k}} \right] \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \mathbf{L}_2 &= R\hat{\mathbf{i}} \times m\mathbf{v}_2, \text{ where } R = \frac{v_i^2 \sin 2\theta}{g} \\ &= mR\hat{\mathbf{i}} \times (v_i \cos \theta \hat{\mathbf{i}} - v_i \sin \theta \hat{\mathbf{j}}) \\ &= -mRv_i \sin \theta \hat{\mathbf{k}} = \left[\frac{-mv_i^3 \sin 2\theta \sin \theta}{g} \hat{\mathbf{k}} \right] \end{aligned}$$

(d) The downward force of gravity exerts a torque in the $-z$ direction.

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

$$\begin{aligned} \text{(a)} \quad \mathbf{r} &= (4.30 \text{ km})\hat{\mathbf{k}} = (4.30 \times 10^3 \text{ m})\hat{\mathbf{k}} \\ \mathbf{p} &= m\mathbf{v} = 12\,000 \text{ kg}(-175\hat{\mathbf{i}} \text{ m/s}) = -2.10 \times 10^6 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s} \\ \mathbf{L} &= \mathbf{r} \times \mathbf{p} = (4.30 \times 10^3 \hat{\mathbf{k}} \text{ m}) \times (-2.10 \times 10^6 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s}) = \left[(-9.03 \times 10^9 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\mathbf{j}} \right] \end{aligned}$$

(b) No. $L = |\mathbf{r}||\mathbf{p}|\sin \theta = mv(r \sin \theta)$, and $r \sin \theta$ is the altitude of the plane. Therefore, $L = \text{constant}$ as the plane moves in level flight with constant velocity.

(c) Zero. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr \sin 180^\circ = 0$.

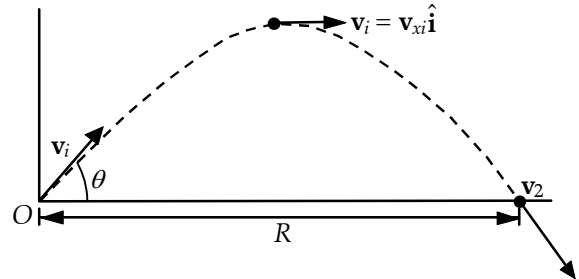


FIG. P11.17

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P11.19 The vector from P to the falling ball is

$$\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2$$

$$\mathbf{r} = (\ell \cos \theta \hat{\mathbf{i}} + \ell \sin \theta \hat{\mathbf{j}}) + 0 - \left(\frac{1}{2} g t^2 \right) \hat{\mathbf{j}}$$

The velocity of the ball is

$$\mathbf{v} = \mathbf{v}_i + \mathbf{a} t = 0 - g t \hat{\mathbf{j}}$$

So $\mathbf{L} = \mathbf{r} \times m \mathbf{v}$

$$\mathbf{L} = m \left[(\ell \cos \theta \hat{\mathbf{i}} + \ell \sin \theta \hat{\mathbf{j}}) + 0 - \left(\frac{1}{2} g t^2 \right) \hat{\mathbf{j}} \right] \times (-g t \hat{\mathbf{j}})$$

$$\mathbf{L} = \boxed{-m \ell g t \cos \theta \hat{\mathbf{k}}}$$

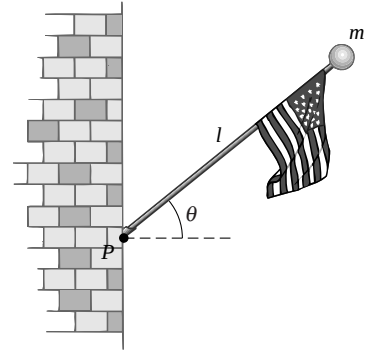


FIG. P11.19

P11.20 In the vertical section of the hose, the water has zero angular momentum about our origin (point O between the fireman's feet). As it leaves the nozzle, a parcel of mass m has angular momentum:

$$L = |\mathbf{r} \times m \mathbf{v}| = m r v \sin 90.0^\circ = m(1.30 \text{ m})(12.5 \text{ m/s})$$

$$L = (16.3 \text{ m}^2/\text{s})m$$

The torque on the hose is the rate of change in angular momentum. Thus,

$$\tau = \frac{dL}{dt} = (16.3 \text{ m}^2/\text{s}) \frac{dm}{dt} = (16.3 \text{ m}^2/\text{s})(6.31 \text{ kg/s}) = \boxed{103 \text{ N} \cdot \text{m}}$$

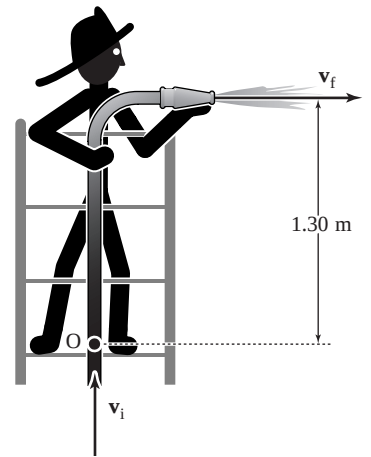


FIG. P11.20

Section 11.3 Angular Momentum of a Rotating Rigid Object

***P11.21** $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{I^2 \omega^2}{I} = \frac{L^2}{2I}$

P11.22 The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5} M R^2 = \frac{2}{5} (15.0 \text{ kg})(0.500 \text{ m})^2 = 1.50 \text{ kg} \cdot \text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I \omega = (1.50 \text{ kg} \cdot \text{m}^2)(3.00 \text{ rad/s}) = 4.50 \text{ kg} \cdot \text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus, $\mathbf{L} = \boxed{(4.50 \text{ kg} \cdot \text{m}^2/\text{s}) \hat{\mathbf{k}}}$.

P11.23 (a) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00 \text{ kg})(0.200 \text{ m})^2(6.00 \text{ rad/s}) = \boxed{0.360 \text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$
 $= \frac{3}{4}(3.00 \text{ kg})(0.200 \text{ m})^2(6.00 \text{ rad/s}) = \boxed{0.540 \text{ kg}\cdot\text{m}^2/\text{s}}$

P11.24 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with $I_h = \frac{m_h L_h^2}{3} = \frac{60.0 \text{ kg}(2.70 \text{ m})^2}{3} = 146 \text{ kg}\cdot\text{m}^2$

and $I_m = \frac{m_m L_m^2}{3} = \frac{100 \text{ kg}(4.50 \text{ m})^2}{3} = 675 \text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi \text{ rad}}{12 \text{ h}} \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 1.45 \times 10^{-4} \text{ rad/s}$

while $\omega_m = \frac{2\pi \text{ rad}}{1 \text{ h}} \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 1.75 \times 10^{-3} \text{ rad/s}$

Thus, $L = 146 \text{ kg}\cdot\text{m}^2(1.45 \times 10^{-4} \text{ rad/s}) + 675 \text{ kg}\cdot\text{m}^2(1.75 \times 10^{-3} \text{ rad/s})$

or $\boxed{L = 1.20 \text{ kg}\cdot\text{m}^2/\text{s}}$

P11.25 (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.1083 \text{ kg}\cdot\text{m}^2$
 $L = I\omega = 0.1083(4.00) = \boxed{0.433 \text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$
 $L = I\omega = 0.433(4.00) = \boxed{1.73 \text{ kg}\cdot\text{m}^2/\text{s}}$

***P11.26** $\sum F_x = ma_x: \quad +f_s = ma_x$

We must use the center of mass as the axis in

$$\sum \tau = I\alpha: \quad F_g(0) - n(77.5 \text{ cm}) + f_s(88 \text{ cm}) = 0$$

$$\sum F_y = ma_y: \quad +n - F_g = 0$$

We combine the equations by substitution:

$$-mg(77.5 \text{ cm}) + ma_x(88 \text{ cm}) = 0$$

$$a_x = \frac{(9.80 \text{ m/s}^2)77.5 \text{ cm}}{88 \text{ cm}} = \boxed{8.63 \text{ m/s}^2}$$

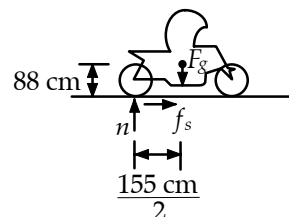


FIG. P11.26

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***P11.27** We require $a_c = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80 \text{ m/s}^2)}{100 \text{ m}}} = 0.313 \text{ rad/s}$$

$$I = Mr^2 = 5 \times 10^4 \text{ kg}(100 \text{ m})^2 = 5 \times 10^8 \text{ kg} \cdot \text{m}^2$$

(a) $L = I\omega = 5 \times 10^8 \text{ kg} \cdot \text{m}^2 \cdot 0.313/\text{s} = \boxed{1.57 \times 10^8 \text{ kg} \cdot \text{m}^2/\text{s}}$

(c) $\sum \tau = I\alpha = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$$\sum \tau \Delta t = I\omega_f - I\omega_i = L_f - L_i$$

This is the angular impulse-angular momentum theorem.

(b) $\Delta t = \frac{L_f - 0}{\sum \tau} = \frac{1.57 \times 10^8 \text{ kg} \cdot \text{m}^2/\text{s}}{2(125 \text{ N})(100 \text{ m})} = \boxed{6.26 \times 10^3 \text{ s}} = 1.74 \text{ h}$

Section 11.4 Conservation of Angular Momentum

P11.28 (a) From conservation of angular momentum for the system of two cylinders:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1} \left(\frac{I_1}{I_1 + I_2}\omega_i \right)^2 = \boxed{\frac{I_1}{I_1 + I_2} \text{ which is less than } 1}.$

P11.29 $I_i\omega_i = I_f\omega_f: (250 \text{ kg} \cdot \text{m}^2)(10.0 \text{ rev/min}) = [250 \text{ kg} \cdot \text{m}^2 + 25.0 \text{ kg}(2.00 \text{ m})^2]\omega_2$

$$\omega_2 = \boxed{7.14 \text{ rev/min}}$$

- P11.30** (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2(mr^2) + 3.00 \text{ kg} \cdot \text{m}^2$$

Before: $r = 1.00 \text{ m}$.

Thus, $I_i = 2(3.00 \text{ kg})(1.00 \text{ m})^2 + 3.00 \text{ kg} \cdot \text{m}^2 = 9.00 \text{ kg} \cdot \text{m}^2$

After: $r = 0.300 \text{ m}$

Thus, $I_f = 2(3.00 \text{ kg})(0.300 \text{ m})^2 + 3.00 \text{ kg} \cdot \text{m}^2 = 3.54 \text{ kg} \cdot \text{m}^2$

We now use conservation of angular momentum.

$$I_f \omega_f = I_i \omega_i$$

or $\omega_f = \left(\frac{I_i}{I_f} \right) \omega_i = \left(\frac{9.00}{3.54} \right) (0.750 \text{ rad/s}) = \boxed{1.91 \text{ rad/s}}$

(b) $K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (9.00 \text{ kg} \cdot \text{m}^2) (0.750 \text{ rad/s})^2 = \boxed{2.53 \text{ J}}$

$$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (3.54 \text{ kg} \cdot \text{m}^2) (1.91 \text{ rad/s})^2 = \boxed{6.44 \text{ J}}$$

- P11.31** (a) Let M = mass of rod and m = mass of each bead. From $I_i \omega_i = I_f \omega_f$, we have

$$\left[\frac{1}{12} M \ell^2 + 2mr_1^2 \right] \omega_i = \left[\frac{1}{12} M \ell^2 + 2mr_2^2 \right] \omega_f$$

When $\ell = 0.500 \text{ m}$, $r_1 = 0.100 \text{ m}$, $r_2 = 0.250 \text{ m}$, and with other values as stated in the problem, we find

$$\omega_f = 9.20 \text{ rad/s}.$$

- (b) Since there is no external torque on the rod,

$$L = \text{constant and } \boxed{\omega \text{ is unchanged}}.$$

- *P11.32** Let M represent the mass of all the ribs together and L the length of each. The original moment of inertia is $\frac{1}{3} ML^2$. The final effective length of each rib is $L \sin 22.5^\circ$ and the final moment of inertia is $\frac{1}{3} M(L \sin 22.5^\circ)^2$ angular momentum of the umbrella is conserved:

$$\frac{1}{3} ML^2 \omega_i = \frac{1}{3} ML^2 \sin^2 22.5^\circ \omega_f$$

$$\omega_f = \frac{1.25 \text{ rad/s}}{\sin^2 22.5^\circ} = \boxed{8.54 \text{ rad/s}}$$

336 Angular Momentum

- P11.33** (a) The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman. From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

$$\text{so, } L_f = I_{\text{woman}} \omega_{\text{woman}} + I_{\text{table}} \omega_{\text{table}} = 0$$

$$\text{and } \omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}} \right) \omega_{\text{woman}} = \left(-\frac{m_{\text{woman}} r^2}{I_{\text{table}}} \right) \left(\frac{v_{\text{woman}}}{r} \right) = -\frac{m_{\text{woman}} r v_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0 \text{ kg}(2.00 \text{ m})(1.50 \text{ m/s})}{500 \text{ kg} \cdot \text{m}^2} = -0.360 \text{ rad/s}$$

$$\text{or } \omega_{\text{table}} = \boxed{0.360 \text{ rad/s (counterclockwise)}}$$

$$(b) \text{ work done} = \Delta K = K_f - 0 = \frac{1}{2} m_{\text{woman}} v_{\text{woman}}^2 + \frac{1}{2} I \omega_{\text{table}}^2$$

$$W = \frac{1}{2} (60 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2} (500 \text{ kg} \cdot \text{m}^2)(0.360 \text{ rad/s})^2 = \boxed{99.9 \text{ J}}$$

- P11.34** When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0 \text{ g}(4.00 \text{ cm} + 6.00 \text{ cm})}{120 \text{ g} + 80.0 \text{ g}} = 4.00 \text{ cm}$$

$$(a) \quad L = r_1 m_1 v_1 + r_2 m_2 v_2 = 0 + (6.00 \times 10^{-2} \text{ m})(80.0 \times 10^{-3} \text{ kg})(1.50 \text{ m/s}) = \boxed{7.20 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}}$$

- (b) The moment of inertia about the CM is

$$\begin{aligned} I &= \left(\frac{1}{2} m_1 r_1^2 + m_1 d_1^2 \right) + \left(\frac{1}{2} m_2 r_2^2 + m_2 d_2^2 \right) \\ I &= \frac{1}{2} (0.120 \text{ kg})(6.00 \times 10^{-2} \text{ m})^2 + (0.120 \text{ kg})(4.00 \times 10^{-2})^2 \\ &\quad + \frac{1}{2} (80.0 \times 10^{-3} \text{ kg})(4.00 \times 10^{-2} \text{ m})^2 + (80.0 \times 10^{-3} \text{ kg})(6.00 \times 10^{-2} \text{ m})^2 \\ I &= 7.60 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}}{7.60 \times 10^{-4} \text{ kg} \cdot \text{m}^2} = \boxed{9.47 \text{ rad/s}}$$

P11.35 (a) $L_i = mv\ell \quad \sum \tau_{\text{ext}} = 0, \text{ so } L_f = L_i = \boxed{mv\ell}$

$$L_f = (m+M)v_f\ell$$

$$v_f = \left(\frac{m}{m+M}\right)v$$

(b) $K_i = \frac{1}{2}mv^2$

$$K_f = \frac{1}{2}(M+m)v_f^2$$

$$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow \text{velocity of the bullet and block}$$

$$\text{Fraction of } K \text{ lost} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{M+m}v^2}{\frac{1}{2}mv^2} = \boxed{\frac{M}{M+m}}$$

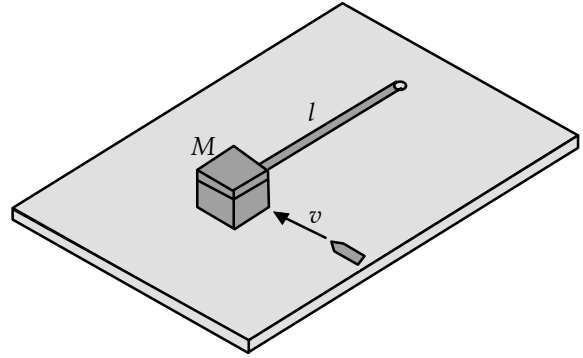


FIG. P11.35

P11.36 For one of the crew,

$$\sum F_r = ma_r : \quad n = \frac{mv^2}{r} = m\omega_i^2 r$$

We require $n = mg$, so $\omega_i = \sqrt{\frac{g}{r}}$

Now, $I_i\omega_i = I_f\omega_f$

$$\left[5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 150 \times 65.0 \text{ kg} \times (100 \text{ m})^2\right] \sqrt{\frac{g}{r}} = \left[5.00 \times 10^8 \text{ kg} \cdot \text{m}^2 + 50 \times 65.0 \text{ kg} (100 \text{ m})^2\right] \omega_f$$

$$\left(\frac{5.98 \times 10^8}{5.32 \times 10^8}\right) \sqrt{\frac{g}{r}} = \omega_f = 1.12 \sqrt{\frac{g}{r}}$$

Now, $|a_r| = \omega_f^2 r = 1.26g = \boxed{12.3 \text{ m/s}^2}$

P11.37 (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

$$L_f = L_i : \quad I\omega = mv_i d$$

or $\left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_i d$

Thus, $\omega = \boxed{\frac{2mv_i d}{(M+2m)R^2}}$

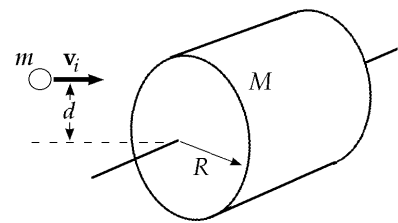


FIG. P11.37

(b) No. Some mechanical energy changes to internal energy in this perfectly inelastic collision.

*P11.38 (a) Let ω be the angular speed of the signboard when it is vertical.

$$\begin{aligned}
 \frac{1}{2}I\omega^2 &= Mgh \\
 \therefore \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 &= Mg\frac{1}{2}L(1 - \cos\theta) \\
 \therefore \omega &= \sqrt{\frac{3g(1 - \cos\theta)}{L}} \\
 &= \sqrt{\frac{3(9.80 \text{ m/s}^2)(1 - \cos 25.0^\circ)}{0.50 \text{ m}}} \\
 &= \boxed{2.35 \text{ rad/s}}
 \end{aligned}$$

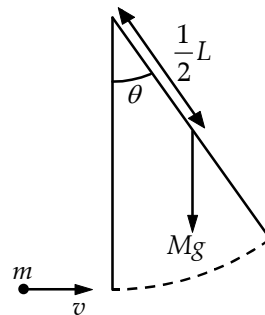


FIG. P11.38

(b) $I_f\omega_f = I_i\omega_i - mvL$ represents angular momentum conservation

$$\begin{aligned}
 \therefore \left(\frac{1}{3}ML^2 + mL^2\right)\omega_f &= \frac{1}{3}ML^2\omega_i - mvL \\
 \therefore \omega_f &= \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L} \\
 &= \frac{\frac{1}{3}(2.40 \text{ kg})(0.5 \text{ m})(2.347 \text{ rad/s}) - (0.4 \text{ kg})(1.6 \text{ m/s})}{\left[\frac{1}{3}(2.40 \text{ kg}) + 0.4 \text{ kg}\right](0.5 \text{ m})} = \boxed{0.498 \text{ rad/s}}
 \end{aligned}$$

(c) Let h_{CM} = distance of center of mass from the axis of rotation.

$$h_{\text{CM}} = \frac{(2.40 \text{ kg})(0.25 \text{ m}) + (0.4 \text{ kg})(0.50 \text{ m})}{2.40 \text{ kg} + 0.4 \text{ kg}} = 0.2857 \text{ m}.$$

Apply conservation of mechanical energy:

$$\begin{aligned}
 (M + m)gh_{\text{CM}}(1 - \cos\theta) &= \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2 \\
 \therefore \theta &= \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M + m)gh_{\text{CM}}}\right] \\
 &= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40 \text{ kg}) + 0.4 \text{ kg}\right](0.50 \text{ m})^2(0.498 \text{ rad/s})^2}{2(2.40 \text{ kg} + 0.4 \text{ kg})(9.80 \text{ m/s}^2)(0.2857 \text{ m})}\right\} \\
 &= \boxed{5.58^\circ}
 \end{aligned}$$

- P11.39** The meteor will slow the rotation of the Earth by the largest amount if its line of motion passes farthest from the Earth's axis. The meteor should be headed west and strike a point on the equator tangentially.

Let the z axis coincide with the axis of the Earth with $+z$ pointing northward. Then, conserving angular momentum about this axis,

$$\sum \mathbf{L}_f = \sum \mathbf{L}_i \Rightarrow I\omega_f = I\omega_i + m\mathbf{v} \times \mathbf{r}$$

$$\text{or} \quad \frac{2}{5}MR^2\omega_f\hat{\mathbf{k}} = \frac{2}{5}MR^2\omega_i\hat{\mathbf{k}} - mvR\hat{\mathbf{k}}$$

$$\text{Thus,} \quad \omega_i - \omega_f = \frac{mvR}{\frac{2}{5}MR^2} = \frac{5mv}{2MR} \quad \text{or}$$

$$\omega_i - \omega_f = \frac{5(3.00 \times 10^{13} \text{ kg})(30.0 \times 10^3 \text{ m/s})}{2(5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})} = 5.91 \times 10^{-14} \text{ rad/s}$$

$$|\Delta\omega_{\max}| \sim 10^{-13} \text{ rad/s}$$

Section 11.5 The Motion of Gyroscopes and Tops

- *P11.40** Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \quad -I_1\omega_1 = I_2 \frac{\theta}{t}$$

$$-20 \text{ kg} \cdot \text{m}^2 (-100 \text{ rad/s}) = 5 \times 10^5 \text{ kg} \cdot \text{m}^2 \left(\frac{30^\circ}{t} \right) \left(\frac{\pi \text{ rad}}{180^\circ} \right)$$

$$t = \frac{2.62 \times 10^5 \text{ s}}{2000} = \boxed{131 \text{ s}}$$

$$\text{*P11.41} \quad I = \frac{2}{5}MR^2 = \frac{2}{5}(5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 = 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2$$

$$L = I\omega = 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2 \left(\frac{2\pi \text{ rad}}{86400 \text{ s}} \right) = 7.06 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

$$\tau = L\omega_p = (7.06 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}) \left(\frac{2\pi \text{ rad}}{2.58 \times 10^4 \text{ yr}} \right) \left(\frac{1 \text{ yr}}{365.25 \text{ d}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) = \boxed{5.45 \times 10^{22} \text{ N} \cdot \text{m}}$$

Section 11.6 Angular Momentum as a Fundamental Quantity

$$\text{P11.42 (a)} \quad L = \frac{h}{2\pi} = mvr \text{ so } v = \frac{h}{2\pi mr} \quad v = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(9.11 \times 10^{-31} \text{ kg})(0.529 \times 10^{-10} \text{ m})} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

$$\text{(b)} \quad K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(2.19 \times 10^6 \text{ m/s})^2 = \boxed{2.18 \times 10^{-18} \text{ J}}$$

$$\text{(c)} \quad \omega = \frac{L}{I} = \frac{\hbar}{mr^2} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(0.529 \times 10^{-10} \text{ m})^2} = \boxed{4.13 \times 10^{16} \text{ rad/s}}$$

Additional Problems

*P11.43 First, we define the following symbols:

I_p = moment of inertia due to mass of people on the equator

I_E = moment of inertia of the Earth alone (without people)

ω = angular velocity of the Earth (due to rotation on its axis)

$T = \frac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)

R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_p \omega_i + I_E \omega_i = (I_p + I_E) \omega_i$$

When the Earth has angular speed ω , the tangential speed of a point on the equator is $v_t = R\omega$.

Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth,

their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_p = \frac{v_p}{R} = \omega + \frac{v}{R}$.

The angular momentum of the system after the people begin to run is

$$L_f = I_p \omega_p + I_E \omega = I_p \left(\omega + \frac{v}{R} \right) + I_E \omega = (I_p + I_E) \omega + \frac{I_p v}{R}.$$

Since no external torques have acted on the system, angular momentum is conserved ($L_f = L_i$),

giving $(I_p + I_E) \omega + \frac{I_p v}{R} = (I_p + I_E) \omega_i$. Thus, the final angular velocity of the Earth is

$$\omega = \omega_i - \frac{I_p v}{(I_p + I_E) R} = \omega_i (1 - x), \text{ where } x \equiv \frac{I_p v}{(I_p + I_E) R \omega_i}.$$

The new length of the day is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_i (1 - x)} = \frac{T_i}{1 - x} \approx T_i (1 + x)$, so the increase in the length of the

day is $\Delta T = T - T_i \approx T_i x = T_i \left[\frac{I_p v}{(I_p + I_E) R \omega_i} \right]$. Since $\omega_i = \frac{2\pi}{T_i}$, this may be written as $\Delta T \approx \frac{T_i^2 I_p v}{2\pi (I_p + I_E) R}$.

To obtain a numeric answer, we compute

$$I_p = m_p R^2 = \left[(5.5 \times 10^9) (70 \text{ kg}) \right] (6.37 \times 10^6 \text{ m})^2 = 1.56 \times 10^{25} \text{ kg} \cdot \text{m}^2$$

and

$$I_E = \frac{2}{5} m_E R^2 = \frac{2}{5} (5.98 \times 10^{24} \text{ kg}) (6.37 \times 10^6 \text{ m})^2 = 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2.$$

$$\text{Thus, } \Delta T \approx \frac{(8.64 \times 10^4 \text{ s})^2 (1.56 \times 10^{25} \text{ kg} \cdot \text{m}^2) (2.5 \text{ m/s})}{2\pi [(1.56 \times 10^{25} + 9.71 \times 10^{37}) \text{ kg} \cdot \text{m}^2] (6.37 \times 10^6 \text{ m})} = \boxed{7.50 \times 10^{-11} \text{ s}}.$$

- *P11.44** (a) $(K + U_s)_A = (K + U_s)_B$
 $0 + mgy_A = \frac{1}{2}mv_B^2 + 0$
 $v_B = \sqrt{2gy_A} = \sqrt{2(9.8 \text{ m/s}^2)6.30 \text{ m}} = \boxed{11.1 \text{ m/s}}$
- (b) $L = mvr = 76 \text{ kg } 11.1 \text{ m/s } 6.3 \text{ m} = \boxed{5.32 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}$ toward you along the axis of the channel.
- (c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.
- (d) $L = mvr$ $v = \frac{5.32 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}{76 \text{ kg } 5.85 \text{ m}} = \boxed{12.0 \text{ m/s}}$
- (e) $(K + U_s)_B + W = (K + U_s)_C$
 $\frac{1}{2}76 \text{ kg}(11.1 \text{ m/s})^2 + 0 + W = \frac{1}{2}76 \text{ kg}(12.0 \text{ m/s})^2 + 76 \text{ kg } 9.8 \text{ m/s}^2 \cdot 0.45 \text{ m}$
 $W = 5.44 \text{ kJ} - 4.69 \text{ kJ} + 335 \text{ J} = \boxed{1.08 \text{ kJ}}$
- (f) $(K + U_s)_C = (K + U_s)_D$
 $\frac{1}{2}76 \text{ kg}(12.0 \text{ m/s})^2 + 0 = \frac{1}{2}76 \text{ kg}v_D^2 + 76 \text{ kg } 9.8 \text{ m/s}^2 \cdot 5.85 \text{ m}$
 $v_D = \boxed{5.34 \text{ m/s}}$
- (g) Let point E be the apex of his flight:
 $(K + U_s)_D = (K + U_s)_E$
 $\frac{1}{2}76 \text{ kg}(5.34 \text{ m/s})^2 + 0 = 0 + 76 \text{ kg}(9.8 \text{ m/s}^2)(y_E - y_D)$
 $(y_E - y_D) = \boxed{1.46 \text{ m}}$
- (h) For the motion between takeoff and touchdown
 $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$
 $-2.34 \text{ m} = 0 + 5.34 \text{ m/s}t - 4.9 \text{ m/s}^2 t^2$
 $t = \frac{-5.34 \pm \sqrt{5.34^2 + 4(4.9)(2.34)}}{-9.8} = \boxed{1.43 \text{ s}}$
- (i) This solution is more accurate. In chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

342 Angular Momentum

P11.45 (a)

$$\begin{aligned}
 I &= \sum m_i r_i^2 \\
 &= m \left(\frac{4d}{3} \right)^2 + m \left(\frac{d}{3} \right)^2 + m \left(\frac{2d}{3} \right)^2 \\
 &= \boxed{7m \frac{d^2}{3}}
 \end{aligned}$$

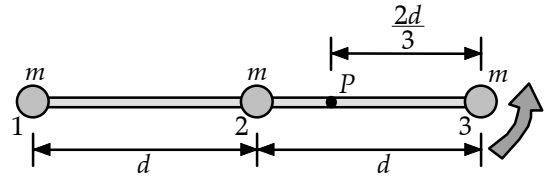


FIG. P11.45

- (b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \left(\frac{d}{3} \right) (-\hat{\mathbf{i}}) \times 3mg(-\hat{\mathbf{j}}) = \boxed{(mgd)\hat{\mathbf{k}}}$$

(c)

$$\alpha = \frac{\tau}{I} = \frac{3mgd}{7md^2} = \boxed{\frac{3g}{7d} \text{ counterclockwise}}$$

(d)

$$a = \alpha r = \left(\frac{3g}{7d} \right) \left(\frac{2d}{3} \right) = \boxed{\frac{2g}{7} \text{ up}}$$

The angular acceleration is not constant, but energy is.

$$\begin{aligned}
 (K + U)_i + \Delta E &= (K + U)_f \\
 0 + (3m)g\left(\frac{d}{3}\right) + 0 &= \frac{1}{2} I \omega_f^2 + 0
 \end{aligned}$$

(e) maximum kinetic energy = $\boxed{mgd}$

(f)

$$\omega_f = \boxed{\sqrt{\frac{6g}{7d}}}$$

(g)

$$L_f = I \omega_f = \frac{7md^2}{3} \sqrt{\frac{6g}{7d}} = \boxed{\left(\frac{14g}{3} \right)^{1/2} md^{3/2}}$$

(h)

$$v_f = \omega_f r = \sqrt{\frac{6g}{7d}} \frac{d}{3} = \boxed{\sqrt{\frac{2gd}{21}}}$$

- P11.46** (a) The radial coordinate of the sliding mass is $r(t) = (0.0125 \text{ m/s})t$. Its angular momentum is

$$L = mr^2\omega = (1.20 \text{ kg})(2.50 \text{ rev/s})(2\pi \text{ rad/rev})(0.0125 \text{ m/s})^2 t^2$$

or $L = (2.95 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}^3)t^2$

The drive motor must supply torque equal to the rate of change of this angular momentum:

$$\tau = \frac{dL}{dt} = (2.95 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}^3)(2t) = \boxed{(0.00589 \text{ W})t}$$

$$(b) \quad \tau_f = (0.00589 \text{ W})(440 \text{ s}) = \boxed{2.59 \text{ N} \cdot \text{m}}$$

$$(c) \quad \mathcal{P} = \tau\omega = (0.00589 \text{ W})t(5\pi \text{ rad/s}) = \boxed{(0.0925 \text{ W/s})t}$$

$$(d) \quad \mathcal{P}_f = (0.0925 \text{ W/s})(440 \text{ s}) = \boxed{40.7 \text{ W}}$$

$$(e) \quad T = m \frac{v^2}{r} = mr\omega^2 = (1.20 \text{ kg})(0.0125 \text{ m/s})t(5\pi \text{ rad/s})^2 = \boxed{(3.70 \text{ N/s})t}$$

$$(f) \quad W = \int_0^{440 \text{ s}} \mathcal{P} dt = \int_0^{440 \text{ s}} (0.0925 \text{ W/s})t dt = \frac{1}{2}(0.0925 \text{ J/s}^2)(440 \text{ s})^2 = \boxed{8.96 \text{ kJ}}$$

- (g) The power the brake injects into the sliding block through the string is

$$\begin{aligned} \mathcal{P}_b &= \mathbf{F} \cdot \mathbf{v} = T v \cos 180^\circ = -(3.70 \text{ N/s})t(0.0125 \text{ m/s}) = -(0.0463 \text{ W/s})t = \frac{dW_b}{dt} \\ W_b &= \int_0^{440 \text{ s}} \mathcal{P}_b dt = - \int_0^{440 \text{ s}} (0.0463 \text{ W/s})t dt \\ &= -\frac{1}{2}(0.0463 \text{ W/s})(440 \text{ s})^2 = \boxed{-4.48 \text{ kJ}} \end{aligned}$$

$$(h) \quad \sum W = W + W_b = 8.96 \text{ kJ} - 4.48 \text{ kJ} = \boxed{4.48 \text{ kJ}}$$

Just half of the work required to increase the angular momentum goes into rotational kinetic energy. The other half becomes internal energy in the brake.

- P11.47** Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \quad \text{or} \quad (mr_a^2)\omega_a = (mr_p^2)\omega_p.$$

Thus, $(mr_a^2)\frac{v_a}{r_a} = (mr_p^2)\frac{v_p}{r_p}$ giving

$$r_a v_a = r_p v_p \quad \text{or} \quad v_a = \frac{r_p}{r_a} v_p = \frac{0.590 \text{ AU}}{35.0 \text{ AU}}(54.0 \text{ km/s}) = \boxed{0.910 \text{ km/s}}.$$

P11.48 (a) $\sum \tau = MgR - MgR = \boxed{0}$

(b) $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, $L = \text{constant}$.

Since the total angular momentum of the system is zero, the
 $\boxed{\text{monkey and bananas move upward with the same speed}}$

at any instant, and $\boxed{\text{he will not reach the bananas}}$ (until they get tangled in the pulley). Also, since the tension in the rope is the same on both sides, Newton's second law applied to the monkey and bananas give the same acceleration upwards.

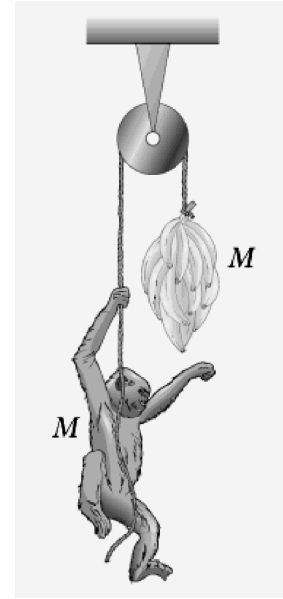


FIG. P11.48

P11.49 (a) $\tau = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}||\mathbf{F}|\sin 180^\circ = 0$

Angular momentum is conserved.

$$L_f = L_i$$

$$mr_i v_i = mrv$$

$$v = \boxed{\frac{r_i v_i}{r}}$$

(b) $T = \frac{mv^2}{r} = \boxed{\frac{m(r_i v_i)^2}{r^3}}$

(c) The work is done by the centripetal force in the *negative* direction.

Method 1:

$$\begin{aligned} W &= \int \mathbf{F} \cdot d\boldsymbol{\ell} = -\int T dr' = -\int_{r_i}^r \frac{m(r_i v_i)^2}{(r')^3} dr' = \left. \frac{m(r_i v_i)^2}{2(r')^2} \right|_{r_i}^r \\ &= \frac{m(r_i v_i)^2}{2} \left(\frac{1}{r^2} - \frac{1}{r_i^2} \right) = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1 \right)} \end{aligned}$$

Method 2:

$$W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2 = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1 \right)}$$

(d) Using the data given, we find

$$v = \boxed{4.50 \text{ m/s}} \quad T = \boxed{10.1 \text{ N}} \quad W = \boxed{0.450 \text{ J}}$$

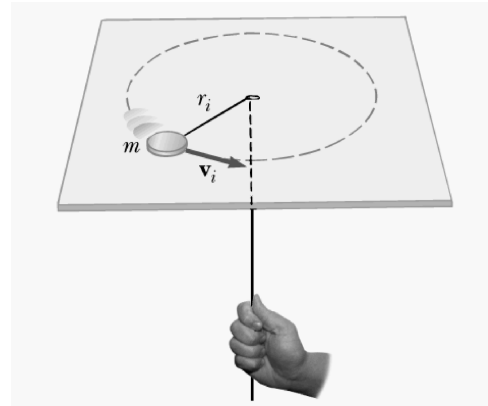


FIG. P11.49

- P11.50** (a) Angular momentum is conserved:

$$\frac{mv_i d}{2} = \left(\frac{1}{12} M d^2 + m \left(\frac{d}{2} \right)^2 \right) \omega$$

$$\omega = \frac{6mv_i}{Md + 3md}$$

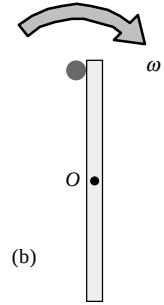
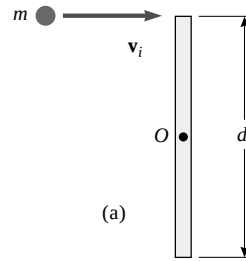


FIG. P11.50

- (b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

$$\frac{1}{2}I\omega^2 = \frac{1}{2} \left(\frac{1}{12} M d^2 + \frac{md^2}{4} \right) \frac{36m^2 v_i^2}{(Md + 3md)^2} = \frac{3m^2 v_i^2 d}{2(Md + 3md)}.$$

The loss of energy is

$$\frac{1}{2}mv_i^2 - \frac{3m^2 v_i^2 d}{2(Md + 3md)} = \frac{mMv_i^2 d}{2(Md + 3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2 d}{2(Md + 3md)mv_i^2} = \frac{M}{M + 3m}.$$

- P11.51** (a) $L_i = m_1 v_{1i} r_{1i} + m_2 v_{2i} r_{2i} = 2mv \left(\frac{d}{2} \right)$
 $L_i = 2(75.0 \text{ kg})(5.00 \text{ m/s})(5.00 \text{ m})$
 $L_i = \boxed{3750 \text{ kg} \cdot \text{m}^2/\text{s}}$

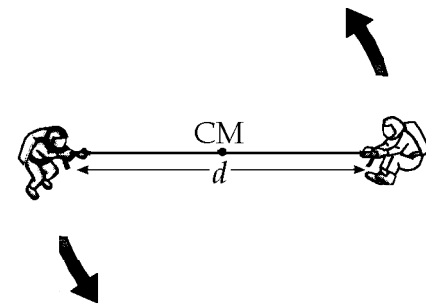


FIG. P11.51

- (b) $K_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$
 $K_i = 2 \left(\frac{1}{2} \right) (75.0 \text{ kg})(5.00 \text{ m/s})^2 = \boxed{1.88 \text{ kJ}}$

- (c) Angular momentum is conserved: $L_f = L_i = \boxed{3750 \text{ kg} \cdot \text{m}^2/\text{s}}$

- (d) $v_f = \frac{L_f}{2(mr_f)} = \frac{3750 \text{ kg} \cdot \text{m}^2/\text{s}}{2(75.0 \text{ kg})(2.50 \text{ m})} = \boxed{10.0 \text{ m/s}}$

- (e) $K_f = 2 \left(\frac{1}{2} \right) (75.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{7.50 \text{ kJ}}$

- (f) $W = K_f - K_i = \boxed{5.62 \text{ kJ}}$

P11.52 (a) $L_i = 2 \left[Mv \left(\frac{d}{2} \right) \right] = \boxed{Mvd}$

(b) $K = 2 \left(\frac{1}{2} Mv^2 \right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2Mr_f} = \frac{Mvd}{2M(\frac{d}{4})} = \boxed{2v}$

(e) $K_f = 2 \left(\frac{1}{2} Mv_f^2 \right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$

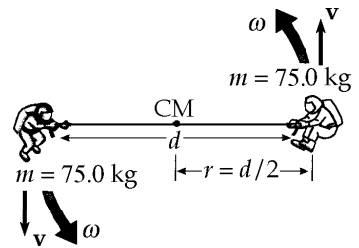


FIG. P11.52

*P11.53 The moment of inertia of the rest of the Earth is

$$I = \frac{2}{5} MR^2 = \frac{2}{5} 5.98 \times 10^{24} \text{ kg} (6.37 \times 10^6 \text{ m})^2 = 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2.$$

For the original ice disks,

$$I = \frac{1}{2} Mr^2 = \frac{1}{2} 2.30 \times 10^{19} \text{ kg} (6 \times 10^5 \text{ m})^2 = 4.14 \times 10^{30} \text{ kg} \cdot \text{m}^2.$$

For the final thin shell of water,

$$I = \frac{2}{3} Mr^2 = \frac{2}{3} 2.30 \times 10^{19} \text{ kg} (6.37 \times 10^6 \text{ m})^2 = 6.22 \times 10^{32} \text{ kg} \cdot \text{m}^2.$$

Conservation of angular momentum for the spinning planet is expressed by $I_i \omega_i = I_f \omega_f$

$$(4.14 \times 10^{30} + 9.71 \times 10^{37}) \frac{2\pi}{86\,400 \text{ s}} = (6.22 \times 10^{32} + 9.71 \times 10^{37}) \frac{2\pi}{(86\,400 \text{ s} + \delta)}$$

$$\left(1 + \frac{\delta}{86\,400 \text{ s}} \right) \left(1 + \frac{4.14 \times 10^{30}}{9.71 \times 10^{37}} \right) = \left(1 + \frac{6.22 \times 10^{32}}{9.71 \times 10^{37}} \right)$$

$$\frac{\delta}{86\,400 \text{ s}} = \frac{6.22 \times 10^{32}}{9.71 \times 10^{37}} - \frac{4.14 \times 10^{30}}{9.71 \times 10^{37}}$$

$$\boxed{\delta = 0.550 \text{ s}}$$

- P11.54** For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a(\sqrt{2} - 1)$.

$$\therefore Mga(\sqrt{2} - 1) = \frac{1}{2} I_{\text{cube}} \omega^2$$

From conservation of angular momentum,

$$\begin{aligned} \frac{4a}{3} mv &= \left(\frac{8Ma^2}{3} \right) \omega \\ \omega &= \frac{mv}{2Ma} \\ \frac{1}{2} \left(\frac{8Ma^2}{3} \right) \frac{m^2 v^2}{4M^2 a^2} &= Mga(\sqrt{2} - 1) \\ v &= \boxed{\frac{M}{m} \sqrt{3ga(\sqrt{2} - 1)}} \end{aligned}$$

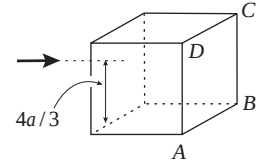
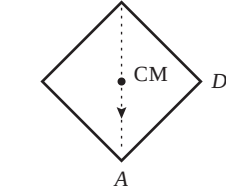


FIG. P11.54

- P11.55** Angular momentum is conserved during the inelastic collision.

$$\begin{aligned} Mva &= I\omega \\ \omega &= \frac{Mva}{I} = \frac{3v}{8a} \end{aligned}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{\text{max}} = a\sqrt{2}$. Using conservation of energy:

$$\begin{aligned} \frac{1}{2} I \omega^2 &= Mg(a\sqrt{2} - a) \\ \frac{1}{2} \left(\frac{8Ma^2}{3} \right) \left(\frac{3v}{8a} \right)^2 &= Mg(a\sqrt{2} - a) \\ v^2 &= \frac{16}{3} ga(\sqrt{2} - 1) \\ v &= \boxed{4 \left[\frac{ga}{3} (\sqrt{2} - 1) \right]^{1/2}} \end{aligned}$$

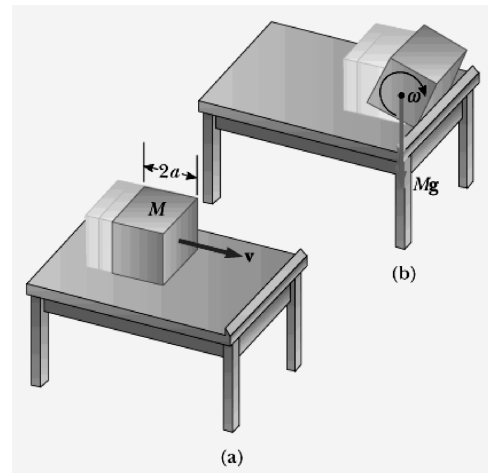


FIG. P11.55

- P11.56** (a) The net torque is zero at the point of contact, so the angular momentum before and after the collision must be equal.

$$\left(\frac{1}{2} MR^2 \right) \omega_i = \left(\frac{1}{2} MR^2 \right) \omega + (MR^2) \omega \quad \omega = \boxed{\frac{\omega_i}{3}}$$

$$(b) \quad \frac{\Delta E}{E} = \frac{\frac{1}{2} \left(\frac{1}{2} MR^2 \right) \left(\frac{\omega_i}{3} \right)^2 + \frac{1}{2} M \left(\frac{R\omega_i}{3} \right)^2 - \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \omega_i^2}{\frac{1}{2} \left(\frac{1}{2} MR^2 \right) \omega_i^2} = \boxed{-\frac{2}{3}}$$

$$\text{P11.57} \quad (a) \quad \Delta t = \frac{\Delta p}{f} = \frac{Mv}{\mu Mg} = \frac{MR\omega}{\mu Mg} = \boxed{\frac{R\omega_i}{3\mu g}}$$

$$(b) \quad W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{18}MR^2\omega_i^2 \quad (\text{See Problem 11.56})$$

$$\mu Mgx = \frac{1}{18}MR^2\omega_i^2 \quad \boxed{x = \frac{R^2\omega_i^2}{18\mu g}}$$

ANSWERS TO EVEN PROBLEMS

$$\text{P11.2} \quad (a) 740 \text{ cm}^2; (b) 59.5 \text{ cm}$$

$$\text{P11.4} \quad (a) 168^\circ; (b) 11.9^\circ \text{ principal value}; \\ (c) \text{ Only the first is unambiguous.}$$

$$\text{P11.6} \quad \text{No; see the solution}$$

$$\text{P11.8} \quad (a) (-7.00 \text{ N}\cdot\text{m})\hat{\mathbf{k}}; (b) (11.0 \text{ N}\cdot\text{m})\hat{\mathbf{k}}$$

$$\text{P11.10} \quad \text{see the solution}$$

$$\text{P11.12} \quad (-22.0 \text{ kg}\cdot\text{m}^2/\text{s})\hat{\mathbf{k}}$$

$$\text{P11.14} \quad \text{see the solution}$$

$$\text{P11.16} \quad (a) 3.14 \text{ N}\cdot\text{m}; (b) (0.400 \text{ kg}\cdot\text{m})v; \\ (c) 7.85 \text{ m/s}^2$$

$$\text{P11.18} \quad (a) (+9.03 \times 10^9 \text{ kg}\cdot\text{m}^2/\text{s}) \text{ south}; (b) \text{ No}; \\ (c) 0$$

$$\text{P11.20} \quad 103 \text{ N}\cdot\text{m}$$

$$\text{P11.22} \quad (4.50 \text{ kg}\cdot\text{m}^2/\text{s}) \text{ up}$$

$$\text{P11.24} \quad 1.20 \text{ kg}\cdot\text{m}^2/\text{s} \text{ perpendicularly into the clock face}$$

$$\text{P11.26} \quad 8.63 \text{ m/s}^2$$

$$\text{P11.28} \quad (a) \frac{I_1\omega_i}{I_1 + I_2}; (b) \frac{K_f}{K_i} = \frac{I_1}{I_1 + I_2}$$

$$\text{P11.30} \quad (a) 1.91 \text{ rad/s}; (b) 2.53 \text{ J}; 6.44 \text{ J}$$

$$\text{P11.32} \quad 8.54 \text{ rad/s}$$

$$\text{P11.34} \quad (a) 7.20 \times 10^{-3} \text{ kg}\cdot\text{m}^2/\text{s}; (b) 9.47 \text{ rad/s}$$

$$\text{P11.36} \quad 12.3 \text{ m/s}^2$$

$$\text{P11.38} \quad (a) 2.35 \text{ rad/s}; (b) 0.498 \text{ rad/s}; (c) 5.58^\circ$$

$$\text{P11.40} \quad 131 \text{ s}$$

$$\text{P11.42} \quad (a) 2.19 \times 10^6 \text{ m/s}; (b) 2.18 \times 10^{-18} \text{ J}; \\ (c) 4.13 \times 10^{16} \text{ rad/s}$$

$$\text{P11.44} \quad (a) 11.1 \text{ m/s}; (b) 5.32 \times 10^3 \text{ kg}\cdot\text{m}^2/\text{s}; \\ (c) \text{ see the solution}; (d) 12.0 \text{ m/s}; \\ (e) 1.08 \text{ kJ}; (f) 5.34 \text{ m/s}; (g) 1.46 \text{ m}; \\ (h) 1.43 \text{ s}; (i) \text{ see the solution}$$

$$\text{P11.46} \quad (a) (0.00589 \text{ W})t; (b) 2.59 \text{ N}\cdot\text{m}; \\ (c) (0.0925 \text{ W/s})t; (d) 40.7 \text{ W}; \\ (e) (3.70 \text{ N/s})t; (f) 8.96 \text{ kJ}; (g) -4.48 \text{ kJ} \\ (h) +4.48 \text{ kJ}$$

$$\text{P11.48} \quad (a) 0; (b) 0; \text{ no}$$

$$\text{P11.50} \quad (a) \frac{6mv_i}{Md + 3md}; (b) \frac{M}{M + 3m}$$

$$\text{P11.52} \quad (a) Mvd; (b) Mv^2; (c) Mvd; (d) 2v; \\ (e) 4Mv^2; (f) 3Mv^2$$

$$\text{P11.54} \quad \frac{M}{m} \sqrt{3ga(\sqrt{2} - 1)}$$

$$\text{P11.56} \quad (a) \frac{\omega_i}{3}; (b) \frac{\Delta E}{E} = -\frac{2}{3}$$